G FACTORISATION OF $ax^2 + bx + c,\ a \neq 1$

In this section we will learn how to factorise quadratic trinomials where the coefficient of x^2 is not 1, and we cannot remove a common factor.

Consider the quadratic trinomial $4x^2 + 11x + 6$.

Using the FOIL rule, we observe that

$$\begin{aligned} &(4x+3)(x+2) \\ &= 4x^2 + 8x + 3x + 6 \\ &= 4x^2 + 11x + 6 \end{aligned}$$

We will now *reverse* the process to factorise $4x^2 + 11x + 6$:

$$\begin{aligned} &4x^2 + 11x + 6 \\ &= 4x^2 + 8x + 3x + 6 && \{\text{`splitting' the middle term}\} \\ &= (4x^2 + 8x) + (3x + 6) && \{\text{grouping in pairs}\} \\ &= 4x(x+2) + 3(x+2) && \{\text{factorising each pair separately}\} \\ &= (4x+3)(x+2) && \{\text{completing the factorisation}\} \end{aligned}$$

But how do we know how to correctly 'split' the middle term? How do we know that $11x$ should be written as $8x + 3x$ rather than $6x + 5x$ or $10x + x$?

The following procedure is used to factorise $ax^2 + bx + c$ by ‘splitting’ the middle term:

Step 1: Find two numbers p and q whose sum is b and whose product is ac.

Step 2: Replace bx by $px + qx$.

Step 3: Complete the factorisation.

Example 15

Self Tutor

Factorise:

a $3x^2 + 17x + 10$

b $6x^2 - 11x - 10$

Example 15 **Self Tutor**

Factorise:

a $3x^2 + 17x + 10$

b $6x^2 - 11x - 10$

a For $3x^2 + 17x + 10$, $ac = 3 \times 10 = 30$ and $b = 17$.

We need two numbers with sum 17 and product 30. These are 2 and 15.

$$\begin{aligned} \therefore \quad 3x^2 + 17x + 10 &= 3x^2 + 2x + 15x + 10 \\ &= x(3x + 2) + 5(3x + 2) \\ &= (3x + 2)(x + 5) \end{aligned}$$

b For $6x^2 - 11x - 10$, $ac = 6 \times -10 = -60$ and $b = -11$.

We need two numbers with sum -11 and product -60. These are -15 and 4.

$$\begin{aligned} \therefore \quad 6x^2 - 11x - 10 &= 6x^2 - 15x + 4x - 10 \\ &= 3x(2x - 5) + 2(2x - 5) \\ &= (2x - 5)(3x + 2) \end{aligned}$$

1 Consider the quadratic trinomial $3x^2 + 7x + 2$.

a Factorise the expression by 'splitting' the middle term into:

- **i** $+6x + x$
- **ii** $+x + 6x$

b Are your factorisations in **a** equivalent?

2 Fully factorise:

- **a** $2x^2 + 5x + 3$
- **b** $2x^2 + 13x + 18$
- **c** $7x^2 + 9x + 2$
- **d** $3x^2 + 13x + 4$
- **e** $3x^2 + 8x + 4$
- **f** $3x^2 + 16x + 21$
- **g** $8x^2 + 14x + 3$
- **h** $21x^2 + 17x + 2$
- **i** $6x^2 + 5x + 1$
- **j** $6x^2 + 19x + 3$
- **k** $10x^2 + 17x + 3$
- **l** $14x^2 + 37x + 5$

3 Consider the quadratic trinomial $4x^2 + 4x - 3$.

a Factorise the expression by 'splitting' the middle term into:

- **i** $+6x - 2x$
- **ii** $-2x + 6x$

b Are your factorisations in **a** equivalent?

4 Fully factorise:

- **a** $2x^2 - 9x - 5$
- **b** $3x^2 + 5x - 2$
- **c** $3x^2 - 5x - 2$
- **d** $2x^2 + 3x - 2$
- **e** $2x^2 + 3x - 5$
- **f** $5x^2 - 8x + 3$
- **g** $11x^2 - 9x - 2$
- **h** $2x^2 - 3x - 9$
- **i** $3x^2 - 17x + 10$
- **j** $5x^2 - 13x - 6$
- **k** $3x^2 + 10x - 8$
- **l** $2x^2 + 17x - 9$
- **m** $2x^2 + 9x - 18$
- **n** $15x^2 + x - 2$
- **o** $21x^2 - 62x - 3$

Example 16 **Self Tutor**

Fully factorise: $-5x^2 - 7x + 6$

Example 16

Self Tutor

Fully factorise: $-5x^2 - 7x + 6$

We remove -1 as a common factor first.

$$\begin{aligned}
& -5x^2 - 7x + 6 \\
&= -1[5x^2 + 7x - 6] \\
&= -[5x^2 + 10x - 3x - 6] \\
&= -[5x(x+2) - 3(x+2)] \\
&= -[(x+2)(5x-3)] \\
&= -(x+2)(5x-3)
\end{aligned}$$

For $5x^2 + 7x - 6$, $ac = -30$ and $b = 7$.
The two numbers with product -30 and sum 7 are 10 and -3.

5 Fully factorise by first removing -1 as a common factor:

a $-3x^2 - x + 14$
b $-5x^2 + 11x - 2$
c $-4x^2 - 9x + 9$
d $-9x^2 + 12x - 4$
e $-8x^2 - 14x - 3$
f $-12x^2 + 16x + 3$

6 **a** Show that $(3x+5)^2 - (2x-3)^2 = 5x^2 + 42x + 16$ by expanding the LHS.

b Factorise $5x^2 + 42x + 16$ by 'splitting' the middle term.

c Factorise $(3x+5)^2 - (2x-3)^2$ using the difference of two squares.

1 **a** **i**
$$\begin{aligned}&3x^2+7x+2\\&=3x^2+6x+x+2\\&=3x(x+2)+1(x+2)\\&=(x+2)(3x+1)\end{aligned}$$

ii
$$\begin{aligned}&3x^2+7x+2\\&=3x^2+x+6x+2\\&=x(3x+1)+2(3x+1)\\&=(3x+1)(x+2)\end{aligned}$$

b Yes, as $ab=ba$.

2 **a** $(2x+3)(x+1)$ **b** $(2x+9)(x+2)$
c $(7x+2)(x+1)$ **d** $(3x+1)(x+4)$
e $(3x+2)(x+2)$ **f** $(3x+7)(x+3)$
g $(4x+1)(2x+3)$ **h** $(7x+1)(3x+2)$
i $(3x+1)(2x+1)$ **j** $(6x+1)(x+3)$
k $(5x+1)(2x+3)$ **l** $(7x+1)(2x+5)$

3 **a** **i** $(2x+3)(2x-1)$ **ii** $(2x-1)(2x+3)$ **b** yes

4 **a** $(2x+1)(x-5)$ **b** $(3x-1)(x+2)$
c $(3x+1)(x-2)$ **d** $(2x-1)(x+2)$
e $(2x+5)(x-1)$ **f** $(5x-3)(x-1)$
g $(11x+2)(x-1)$ **h** $(2x+3)(x-3)$
i $(3x-2)(x-5)$ **j** $(5x+2)(x-3)$
k $(3x-2)(x+4)$ **l** $(2x-1)(x+9)$
m $(2x-3)(x+6)$ **n** $(5x+2)(3x-1)$
o $(21x+1)(x-3)$

5 **a** $-(3x+7)(x-2)$ **b** $-(5x-1)(x-2)$
c $-(4x-3)(x+3)$ **d** $-(3x-2)^2$
e $-(4x+1)(2x+3)$ **f** $-(6x+1)(2x-3)$

6 **a**
$$\begin{aligned}&(3x+5)^2-(2x-3)^2\\&=9x^2+30x+25-[4x^2-12x+9]\\&=9x^2+30x+25-4x^2+12x-9\\&=5x^2+42x+16\end{aligned}$$

b $(5x+2)(x+8)$
$\{ac=80,\ b=42,\ \text{split is } +40x+2x\}$

c
$$\begin{aligned}&(3x+5)^2-(2x-3)^2\\&=(3x+5+2x-3)(3x+5-2x+3)\\&=(5x+2)(x+8)\end{aligned}$$